

Effective Field Theories from QCD*

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Abstract

We present a method for extracting effective Lagrangians from QCD. The resulting effective Lagrangians are based on exact rewrites of cut-off QCD in terms of these new collective field degrees of freedom. These cut-off Lagrangians are thus “effective” in the sense that they explicitly contain some of the physical long-distance degrees of freedom from the outset. As an example we discuss the introduction of a new collective field carrying the quantum numbers of the η' -meson.

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1 Introduction

One possible test for low-energy QCD would be a direct calculation of the parameters of chiral perturbation theory. Of course, we can not provide the solution of such an ambitious program. However, we will demonstrate a method for doing the first step in this direction. We show how to introduce the appropriate effective degrees of freedom and how to extract a recipe for determining the parameters of a suitable effective Lagrangian. For technical reasons we concentrate on the simpler Abelian case of an effective field carrying the quantum numbers of a flavour singlet pseudoscalar meson. Such an effective field should describe the dynamics of the η' -meson. Finally, we outline the extension of this method to the more interesting non-Abelian case resulting in an effective Lagrangian analogous to the one of chiral perturbation theory.

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Our main tool is the *gauge-symmetric collective field technique* [1, 2]. It was used to show that (1+1)-dimensional *bosonization* (and fermionization) [3] are only two extremes of a continuum of equivalent field theory descriptions [2, 4]. In fact, both can be viewed as particular *gauge fixings* of a “higher” gauge-symmetric theory that contains both bosons and fermions [4]. The extent to which this method can be applied to QCD has recently been discussed in ref. [5], and the purpose of this talk is to provide a short review of that work.

2 The η' -meson from QCD

2.1 From QCD to “super”-QCD

Our starting point is a generating functional for QCD with N_f flavours in the chiral limit of vanishing quark masses. In Euclidean space-time it is of the form

$$\begin{aligned}\mathcal{Z}_{QCD}[A] &= \int \mathcal{D}[\bar{\psi}, \psi] \mathcal{D}\mu[G] e^{-\int d^4x \mathcal{L}_{QCD}} \\ \mathcal{L}_{QCD} &= \bar{\psi}(x)(\not{D} - i\not{A}(x)\gamma_5)\psi(x) \\ &\quad + \frac{1}{4g^2} \text{tr} G_{\mu\nu}(x) G_{\mu\nu}(x) .\end{aligned}\tag{1}$$

The external Abelian axial vector source $A_\mu(x)$ serves to define appropriate Green functions through functional differentiation. The $SU(N_c)$ -valued covariant derivative is denoted by D_μ and $G_{\mu\nu}(x)$ is the corresponding field strength tensor. The gauge-fixing terms of the usual $SU(N_c)$ colour gauge symmetry are implicitly part of the gluon measure $\mathcal{D}\mu[G]$. For the following we only have to specify explicitly a regularization for the fermionic sector. A convenient consistent scheme in this sector is provided by a set of Pauli-Villars regulator fields [6].

As the field transformation introducing a pseudoscalar field $\theta(x)$ we choose a chiral rotation of the quark fields:

$$\psi(x) = e^{i\theta(x)\gamma_5} \chi(x) \quad , \quad \bar{\psi}(x) = \bar{\chi}(x) e^{i\theta(x)\gamma_5}\tag{2}$$

In order to promote $\theta(x)$ into a dynamical field we integrate over all possible configurations. The result is the generating functional of what one may call “super”-QCD:

$$\begin{aligned}\mathcal{Z}_{SQCD}[A] &= \int \mathcal{D}[\theta] \mathcal{D}[\bar{\chi}, \chi] \mathcal{D}\mu[G] e^{-\int d^4x \mathcal{L}_{SQCD}} \\ \mathcal{L}_{SQCD} &= \mathcal{L}_{QCD} + i\bar{\chi} \not{\partial} \theta \gamma_5 \chi \\ &\quad + \frac{N_f}{2} \partial_\mu \theta f^2 \partial_\mu \theta - N_f A_\mu f^2 \partial_\mu \theta \\ &\quad - 2iN_f \theta Q + \dots .\end{aligned}\tag{3}$$

The terms in the last two lines arise from the Jacobian of the transformation [6]. Only the leading terms of an expansion in powers of $\partial_\mu \theta$ and A_μ are shown. The term in the last line is due to the $U(1)$ -anomaly, with the instanton density

$$Q = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{tr} G_{\mu\nu} G_{\rho\sigma} \quad . \quad (4)$$

Assuming that the gluonic measure implies a summation over all integer instanton numbers N_{inst} one immediately realizes that $\theta(x)$ is globally periodic, *i.e.*,

$$\theta(x) \equiv \theta(x) + \frac{n\pi}{N_f} \quad . \quad (5)$$

Thus, one can restrict the θ -integration globally to the interval $[0, \pi/N_f]$.

The object f^2 in eq. (3) is a nonlocal operator:

$$\begin{aligned} f^2 = & -\frac{N_c \kappa_2 \Lambda^2}{2\pi^2} + \frac{N_c}{12\pi^2} \partial^2 - \frac{N_c \kappa_{-2}}{24\pi^2 \Lambda^2} \partial^2 \partial^2 \\ & - \frac{\kappa_{-2}}{24\pi^2 \Lambda^2} \text{tr}_c G_{\nu\rho} G_{\nu\rho} + \dots \end{aligned} \quad (6)$$

For simplicity we have displayed the leading terms of a gradient expansion. The coefficients κ_2, κ_{-2} in eq.(6) are regularization-scheme dependent constants [5]. There are two important features of f^2 :

- It induces a higher-derivative (or essentially Pauli-Villars) *regularized* bosonic propagator with a regulator mass proportional to Λ^2 , at least in a perturbative sense.
- The possible occurrence of gluonic condensates in (6) signals spontaneous chiral symmetry breaking. However, in the presence of gluonic condensates the implied expansion in inverse powers of the cutoff may fail to converge and one has to live with the full nonlocal structure of f^2 .

Finally, as a consequence of the integration over the collective field $\theta(x)$ a chiral gauge symmetry appears [2]:

$$\begin{aligned} \chi(x) & \rightarrow e^{i\alpha(x)\gamma_5} \chi(x) \\ \bar{\chi}(x) & \rightarrow \bar{\chi}(x) e^{i\alpha(x)\gamma_5} \\ \theta(x) & \rightarrow \theta(x) - \alpha(x). \end{aligned} \quad (7)$$

It is this gauge symmetry which has to be removed by suitable gauge fixing terms in order to leave the generating functional $Z_{QCD}[A]$ unaltered.

2.2 A partial bosonization

For that purpose we consider the change in the divergence of the axial singlet current:

$$i\partial_\mu \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle = i\partial_\mu \langle \bar{\chi} \gamma_\mu \gamma_5 \chi \rangle + N_f \partial_\mu \langle f^2 \partial_\mu \theta \rangle + \dots \quad (8)$$

The additional terms represented by dots are at least of third order in $\theta(x)$. In order to saturate the anomalous chiral Ward identities by the field $\theta(x)$ we have to eliminate the χ -dependent part of eq. (8). The resulting gauge fixing function $\Phi(x)$ reads:

$$\Phi = i \frac{\partial_\mu}{N_f f_0^2 \partial^2} \bar{\chi} \gamma_\mu \gamma_5 \chi . \quad (9)$$

For simplicity we have replaced f^2 by a constant f_0^2 to be explained later on. The function Φ is defined in a very formal manner on account of the inverse Laplacian. Its presence ensures the proper gauge fixing of global chiral gauge transformations.

We now follow the standard Faddeev-Popov procedure introducing a field b ensuring the gauge constraint $\Phi(x) = 0$ as well as Grassmannian ghosts \bar{c} and c . The original generating functional $Z_{QCD}[A]$ now appears as

$$\begin{aligned} \mathcal{Z}_{QCD}[A] &= \int \mathcal{D}[\theta] \mathcal{D}[\bar{\chi}, \chi] \mathcal{D}\mu[G] \mathcal{D}[b, c, \bar{c}] \\ &\quad e^{-\int d^4x \mathcal{L}'} \\ \mathcal{L}' &= \mathcal{L}_{SQCD} + \frac{i}{N_f f_0^2} \bar{\chi} \not{B} \gamma_5 \chi + \bar{c} c + \dots \end{aligned} \quad (10)$$

The field b is related to the purely longitudinal axial vector field B_μ by the relation

$$b(x) = \partial_\mu B_\mu(x) . \quad (11)$$

The global periodicity of the chiral gauge transformation implies that the measure for b extends over an infinite number of topological sectors:

$$\int \mathcal{D}[b] = \sum_{k=-\infty}^{+\infty} \int \mathcal{D}[b]_k \quad (12)$$

For each integer k the field b obeys the topological constraint

$$\int d^4x \, b(x) = \int d^4x \, \partial_\mu B_\mu(x) = ikN_f . \quad (13)$$

The dots in \mathcal{L}' in (10) indicate higher order terms in b which are necessary due to the fact that the gauge fixing term itself modifies the regularized fermionic measure.

2.3 The physical content of $\theta(x)$

The field $\theta(x)$ saturates the chiral Ward identities for the divergence of the axial singlet current by construction. The spectrum of this operator, which includes the η' , is now described by θ . Just in order to illustrate how non-trivial results can be extracted from the effective Lagrangian \mathcal{L}' in (10), let us integrate out all fields in (10) except θ to arrive at an effective Lagrangian

$$\mathcal{L}_{eff} = \frac{F_0^2}{2} \partial_\mu \theta \partial_\mu \theta + \frac{F_0^2 M_0^2}{2} \theta^2 + \dots \quad (14)$$

The dots denote higher derivative terms and self-interactions of order θ^3 . The parameters F_0 and M_0 are defined through

$$F_0^2 M_0^2 = 4N_f^2 \int d^4x \langle Q(x) Q(0) \rangle_{trunc} \quad (15)$$

and

$$F_0^2 = N_f f_0^2 - \frac{N_f^2}{2} \int d^4x x^2 \langle Q(x) Q(0) \rangle_{trunc} \quad , \quad (16)$$

where f_0^2 is just the zero-momentum limit of $\langle f^2 \rangle_{trunc}$.

The identification of F_0 and M_0 with the decay constant and mass of the η' -meson can be made only in the case where this meson is substantially lighter than other excitations with the same quantum numbers. This is the case in the limit $N_c \rightarrow \infty$ where (15) reduces to the relation derived by Witten [7] and Veneziano [8]. However, the expectation values $\langle \dots \rangle_{trunc}$ have to be taken with respect to a “truncated” version of QCD:

$$\mathcal{L}_{trunc} = \mathcal{L}_{SQCD} + \frac{i}{N_f f_0^2} \bar{\chi} \not{B} \gamma_5 \chi + \bar{c} c + \dots \quad (17)$$

Witten [7] argued that in the large- N_c limit the mass of the η' can be derived from the topological susceptibility of pure Yang-Mills theory, *i.e.* QCD without quarks. Here, we are not removing the quarks completely. It is the topological property of the Lagrange-multiplier field b which provides a nonvanishing topological susceptibility and thus a mass for the η' . The reason is that fermionic zero modes in the presence of topological nontrivial fields imply a constraint

$$N_{inst} + k = 0 \quad (18)$$

As we sum over k we also cover all instanton sectors.

We could go beyond the large- N_c limit if we skip the gradient expansion leading to the effective Lagrangian (14). Then we can still derive the propagator for θ containing contributions from all possible excitations with the quantum numbers of the η' .

3 The non-Abelian case: An outline

Effective fields for the $SU(N_f)$ pseudoscalar multiplet can be introduced along similar lines. After a suitable chiral transformation one has to fix a gauge. One can again use the saturation of chiral Ward identities by the new fields as the guiding demand. Integrating out all fields except the pseudoscalars followed by a gradient expansion should result in an effective Lagrangian analogous to the one of chiral perturbation theory. Within such an approach one could determine relations between the parameters of the resulting effective Lagrangian. An interesting alternative would be the application of this approach to lattice-QCD. The calculation of the parameters of chiral perturbation theory could then be done numerically. This would require relatively small lattices due to the absence of Goldstone-bosons in the truncated theory.

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